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GEOMETRICAL OPTICS FOR GYROTROPIC BODIES

by

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REPORT

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Submitted by	W. C. Y. Lee, L. Peters, Jr., and C.H. Walter Antenna Laboratory Department of Electrical Engineering
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ABSTRACT

The methods of geometrical optics are extended so that they may be applied to gyrotropic bodies. Various internal and external reflections are considered at non parallel planar interfaces and means of determining the ray path or directions of energy flow are derived. Non-planar geometrics may be represented by the tangent planes at the various points of incidence. A method is given for computing the phases of the various fields. These values may be used to determine the reflected fields from such a gyrotropic body.

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GEOMETRICAL OPTICS FOR GYROTROPIC BODIES

I. INTRODUCTION

A modified geometrical optics method has been developed which may be used to determine the electromagnetic scattering properties of finite, homogeneous, isotropic bodies. The dielectric sphere and cylinder have been used to illustrate the method[1] but is is not restricted to such shapes. It has been postulated that these same methods may be applied to obtain the electromagnetic scattering properties for the gyrotropic case. Similar methods would also apply to any anisotropic body. However, the usual form of geometrical optics or ray methods essential to the application of the modified geometrical optics method is restricated to the isotropic media.

A version of geometrical optics is presented which may be used to apply the modified geometrical optics method to gyrotropic bodies. It has been shown for the isotropic case that the reflection and transmission coefficients derived using an infinite planar geometry apply remarkably well to curved surfaces with radii of curvature as small as 0.1λ. Such coefficients are thus considered to be valid, with the exception of the region of Rayleigh scattering.

Reflection and transmission coefficients at a sharp boundary have been obtained by Budden[2] and Wait[3], and numerical data presented by Yabroff[4]. Unz has used a boundary value solution to study the more general problem of transmission and reflection at the planar boundary between two semi-finite gyrotropic media[5], and also to study the transmission and reflection due to an infinite gyrotropic slab[6]. Since there are generally two reflected waves and two transmitted waves when a plane wave is incident from free space on an anisotropic medium interface, there are two reflection coefficients and two transmissions coefficients. These coefficients are different for the two principle polarizations (parallel and perpendicular) as is true for the isotropic case. The propagation constants in the gyrotropic medium for two waves can be found from Snells law

$$\sin \theta_I = n_1 \sin \theta_{p_1} = n_2 \sin \theta_{p_2},$$

where θ_I is the angle between the wave normal and the normal to the interface (see Fig. 1).

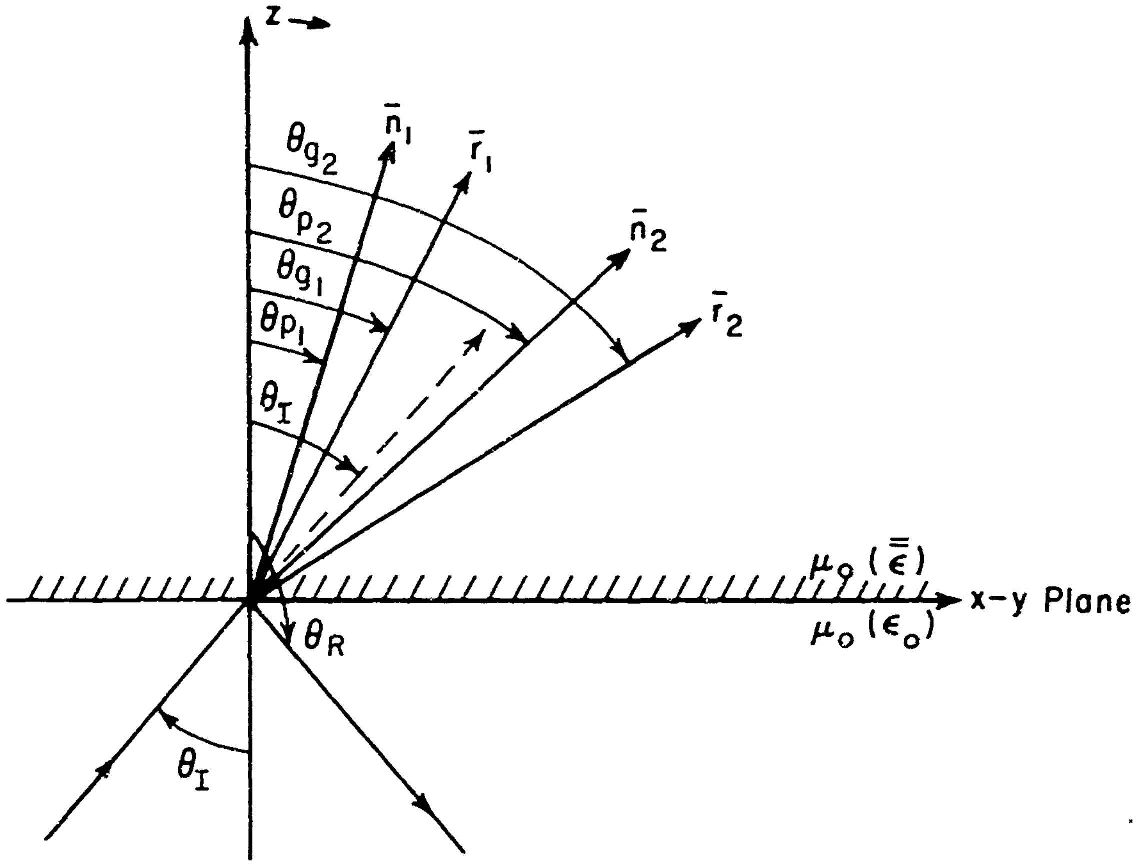

Fig. 1. Plane wave incident on a plasma gyrotropic interface.

Consider a plane wave incident upon the gyrotropic medium from free space, as illustrated in Fig. 1. The two refractive indices, n_1 and n_2, of two waves in the gyrotropic medium could be found from the above equation if θ_{p_1} and θ_{p_2} are known; but the values of θ_{p_1} and θ_{p_2} depend on the orientation of the static magnetic field $\bar{Y}(Y_L, Y_T)$, which in turn depends on n_1 and n_2. Solving Maxwell's equations with the susceptibility matrix derived from the constitutive relation (see Appendix II) yields the Booker Quartic equation (see Appendix I). This equation may be factored to find the component of the propagation factor in the vertical direction (i.e., normal to the interface). The other components of the propagation factor are those of the incident wave due to the requirement that tangential fields be matched at the boundary. Thus the major step in obtaining the direction of the wave normal consists of factoring this well-known quartic equation.

The four distinct roots of the Booker quartic equation are associated with the angle of incidence. Two of these roots are associated with the so-called upgoing waves and the other two with downgoing waves. Two upgoing (or downgoing) waves incident from the gyrotropic medium

on the "gyrotropic medium-free space" interface sets up two downgoing (or upgoing) reflected waves in the medium and two transmitted waves into free space. Reflection at a second interface not parallel to the original one requires that Booker's equation again be solved. However, a transformation of coordinates yields the appropriate values of n, θ_p for the incident wave and thus reduces the complexity of the solution.

The use of non-parallel interfaces represents the appropriate approximation for determining the angle of refraction and the reflection and transmission coefficients for two- or three-dimensional bodies with curved surfaces, such as the cylinder or the sphere. The tangent planes at the various points of incidence provide the appropriate interfaces which are used to represent the body to determine these parameters, as has been demonstrated for the isotropic body[1].

Determination of the refractive indices, the reflection and transmission coefficients, and the direction of the wave normals does not, however, complete the geometrical optics formulation. It is also necessary that the direction of energy flow, herein designated as the ray path, must also be determined, since, in general, it does not coincide with the wave normal. Application of the concept of stationary phase yields a relatively simple expression for the ray path.

Conservation of energy now readily yields the magnitude of the fields, with respect to some reference point associated with any diverging ray system within the usual restrictions of geometrical optics. The phase of these fields (ϕ) is readily obtained by using the component of the vector propagation constant ($\overline{n}$) in the direction of the ray path ($\overline{r}$), i.e., $\phi = \overline{n} \cdot \overline{r}$. Thus the fields may be written as

$$\mu(\ell) = A_o \, e^{j\phi_o} \, F(\ell) \, e^{-jk\overline{n} \cdot \overline{r}} ,$$

where

$A_o \, e^{j\phi_o}$ is the field at the reference point,

$F(\ell)$ is the spatial attenuation factor obtained from the conservation of energy, and

ℓ is the distance from the reference point along the vector $\overline{r}$ (where $\ell = r$ in Fig. 4, assuming o as a reference point).

The above equation will be discussed in Section II-D.

II. WAVE MECHANISM IN A GENERAL GYROTROPIC MEDIUM

A. Wave Properties in a General Gyrotropic Medium

Consider the case of a plane electromagnetic wave propagating in a gyrotropic medium at some arbitrary angle, θ, with respect to the positive z axis. The usual spherical coordinate system is used (see Fig. 1) and $e^{j\omega t}$ time convention is assumed. Then

$$(1) \qquad e^{-jk\overline{n}\cdot\overline{r}} = e^{-jk(S_1x + S_2y + qz)},$$

where

$|n| = \frac{c}{v}$ is the refractive index,

$k = \frac{\omega}{c}$,

c = velocity of light in free space,

$q = n \cos \theta_p$,

$\overline{n}$ = vector parallel with axis of wave normal, and

$\overline{r}$ = vector from origin to the point (x,y,z);

and

$$(2) \qquad n^2 = S_1^2 + S_2^2 + q^2.$$

The parameter q is the solution of a fourth order algebraic equation known as "Booker's quartic"[7,8] equation and has four roots. This equation is given in Appendix A. Two of its roots (q_1, q_2) are characterized by

$$\mathrm{Im}(q_1) < 0 \qquad \text{and} \qquad \mathrm{Im}(q_2) < 0 .$$

These give waves propagating in the +z direction and are known as "upgoing" waves[9]. The other pair, q_3, q_4, are such that

$$\mathrm{Im}(q_3) > 0, \qquad \mathrm{Im}(q_4) > 0 ,$$

and propagate in the -z direction; consequently, they are known as "downgoing" waves. This value of Im(q) indicates the direction of energy flow, while the value of Re(q) indicates the direction of the phase velocity vector or the direction of the wave normal. The paramaters S_1 and S_2 satisfy

(3a) $$\sqrt{S_1^2 + S_2^2} = n \sin \theta_p ,$$

(3b) $$\cos \tau' = \frac{S_1}{n}, \quad \cos \tau'' = \frac{S_2}{n}, \text{ and } \quad \cos \theta_p = \frac{q}{n} ,$$

where τ', and τ'', and θ_p are the angles between the wave normal and the respective coordinate axes (x, y, z). The angle of the wave normal, θ, shown in Fig. 1 is readily obtained from Eqs. (3a) and (3b) when Booker's quartic equation has been factored, i.e.,

(4) $$\theta_p = \tan^{-1} \frac{\sqrt{S_1^2 + S_2^2}}{q} .$$

In order to set up boundary conditions in as simple a manner as is practicable, it is necessary to express all of the fields in terms of one field component, E_z, in the manner developed by Unz[5]. This is done by first writing Maxwell's equations for the gyrotropic media as

(5) $$[S]\eta_o\overline{H} = - [I]\overline{E} - \frac{1}{\epsilon_o}\overline{P} \text{ and}$$

(6) $$[S]\overline{E} = \eta_o\overline{H},$$

where

$$[S] = \begin{bmatrix} 0 & -q & S_2 \\ q & 0 & -S_1 \\ -S_2 & S_1 & 0 \end{bmatrix} ; \eta_o = \sqrt{\frac{\mu_o}{\epsilon_o}}$$

and

[I] is the identity matrix.

The constitutive relation

(7) $$\frac{1}{\epsilon_o}\overline{P} = [M]\overline{E}$$

relates $\overline{P}$ and $\overline{E}$, where $[M]$ is the susceptibility matrix. The matrices $[S]$ and $[M]$ are given in Appendix II as developed by Budden[10].

Substituting Eqs. (6) and (7) into Eq. (5) yields

(8) $$\{[S]\,[S] + [I] + [M]\}\,\overline{E} = 0.$$

Equation (8) is equivalent to three homogeneous equations with three field components, E_x, E_y, and E_z. Expressing E_x and E_y in terms of E_z, from Eq. (8),

$$(1 - q^2 - S_2^2 + M_{xx})E_x + (S_1 S_2 + M_{xy})E_y = -(S_1 q + M_{xz})E_z \text{ and}$$

$$(S_1 S_2 + M_{yx})E_x + (1 - q^2 - S_1^2 + M_{yy})E_y = -(S_2 q + M_{yz})E_z.$$

Then

(9a) $$E_x = \pi_x E_z = \frac{-(S_1 q + M_{xz})(1 - q^2 - S_1^2 + M_{yy}) + (S_1S_2 + M_{xy})(S_2 q + M_{yz})}{(1 - q^2 - S_2^2 + M_{xx})(1 - q^2 - S_1^2 + M_{yy}) - (S_1S_2 + M_{xy})(S_1S_2 + M_{yx})}\, E_z$$

and

(9b) $$E_y = \pi_y E_z = \frac{-(1 - q^2 - S_2^2 + M_{xx})(S_2 q + M_{yz}) + (S_1 q + M_{xz})(S_1S_2 + M_{yx})}{(1 - q^2 - S_2^2 + M_{xx})(1 - q^2 - S_1^2 + M_{yy}) - (S_1S_2 + M_{xy})(S_1S_2 + M_{yx})}\, E_z.$$

From Eq. (6) one may obtain

(9c) $$H_x = \frac{\eta_x}{\eta_o} E_z = \frac{S_2 - q\pi_y}{\eta_o}\, E_z \,,$$

(9d) $$H_y = \frac{\eta_y}{\eta_o} E_z = \frac{-S_1 + q\pi_x}{\eta_o}\, E_z \,, \text{ and}$$

(9e) $$H_z = \frac{\eta_z}{\eta_o} E_z = \frac{-S_2\pi_x + S_1\pi_y}{\eta_o} E_z .$$

For each value of q (i.e., q_1, q_2, q_3, q_4), one may obtain a set of values as π_x, π_y, η_x, η_y η_z, E_z, E_x, E_y, H_x, H_y, and H_z.

Unz has applied equations of the above type to the boundary between the gyrotropic medium and free space to obtain various reflection and transmission coefficients for various interfaces[5]. In particular, he has obtained the reflection and transmission coefficients for plane wave incidence from free space up to gyrotropic media, from gyrotropic media up to free space, and from gyrotropic media down to free space. Without loss of generality, the coordinate system is chosen so that $S_1 = S$ and $S_2 = 0$. This reduces the complexity of the solution. These coefficients are tabulated in Appendix III for the convenience of the reader.

B. Direction of Ray Path

One method of obtaining the direction of the ray path consists of computing the poynting vector in the anisotropic medium. A much simpler method, given by Budden[10], consists of the application of the concept of stationary phase to compute the direction in which the energy is traveling.

The phase of the electromagnetic wave in the anisotropic medium is given by

(10) $$\phi = k(S_1 x + S_2 y + qz).$$

The phase is stationary when its first partial derivative vanishes or

(11a) $$\frac{\partial \phi}{\partial S_1} = k\left(x + \frac{\partial q}{\partial S_1} z\right) = 0$$

and

(11b) $$\frac{\partial \phi}{\partial S_2} = k\left(y + \frac{\partial q}{\partial S_2} z\right) = 0.$$

The ray path is therefore along the direction that satisfies Eqs. (11) or

(12a) $$\tan \theta_{g_{s_1}} = \frac{x}{z} = -\frac{\partial q}{\partial S_1}$$

and

(12b) $$\tan \theta_{g_{s_2}} = \frac{y}{z} = -\frac{\partial q}{\partial S_2} ,$$

where θ_g is the direction of the ray path measured from the positive z axis (the normal to the interface as shown in Fig. 1).

Without loss of generality, the coordinate system is chosen as before, so that $S_1 = S$ and $S_2 = 0$. Equation (12) then becomes

(13a) $$\tan \theta_{g_{s_1}} = -\left.\frac{\partial q}{\partial S_1}\right|_{S_1 = S}$$

and

(13b) $$\tan \theta_{g_{s_2}} = -\left.\frac{\partial q}{\partial S_2}\right|_{S_2 = 0} .$$

Booker's quartic equation is

(14) $$F(\ell) = \alpha\, q^4 + \beta q^3 + \gamma q^2 + \delta q + \epsilon = 0 ,$$

where the coefficients are functions of S_1 and S_2. For the lossless case, the collision frequency is negligible and

(14a) $$\alpha = (1-Y^2) + X(Y_z{}^2-1) = (1-Y^2) + X(\ell_1{}^2 Y^2-1) ,$$

(14b) $$\beta = 2X(S_1 Y_z Y_x + S_2 Y_z Y_y) = 2\ell_3 XY^2(S_1\ell_1 + S_2\ell_2) ,$$

(14c) $$\gamma = -2(1-X)(C^2-X) + 2Y^2(C^2-X) + X[Y^2 - CY_z{}^2 + (S_1 Y_x + S_2 Y_y)^2]$$

$$= -2(1-X)(C^2-X) + 2Y^2(C^2-X) + XY^2[1 - C^2\ell_3{}^2 + (S_1\ell_1 + S_2\ell_2)^2],$$

(14d) $\delta = -2C^2X(S_1Y_zY_x+S_2Y_zY_y) = -2C^2\ell_3XY^2(S_1\ell_1+S_2\ell_2)$,

and

(14e) $\epsilon = (1-X)(C^2-X^2)^2 - C^2Y^2(C^2-X) - C^2X(S_1Y_x+S_2Y_y)^2$

$= (1-X)(C^2-X)^2 - C^2Y^2(C^2-X) - C^2XY^2(S_1\ell_1+S_2\ell_2)^2$,

where ℓ_1, ℓ_2, and ℓ_3 are the direction cosines of the $\overline{Y}$ vector with respect to the x, y, z axes.

Equations (14) are valid for all incident angles or all values of S. Thus

(15) $$\frac{dF(q)}{dS_1} = \frac{dF(q)}{dS_2} = 0 \ .$$

Differentiating Eq. (14) with respect to S_1 yields

(16) $$\frac{dF(q)}{dS_1} = \frac{\partial F(q)}{\partial q}\frac{\partial q}{\partial S_1} + \frac{\partial \alpha}{\partial S_1}q^4 + \frac{\partial \beta}{\partial S_1}q^3 + \frac{\partial \gamma}{\partial S_1}q^2 + \frac{\partial \delta}{\partial S_1}q + \frac{\partial \epsilon}{\partial S_1} = 0 \ .$$

There is a similar equation for $\frac{dF(q)}{dS_2}$ in which S_1 is replaced by S_2 . Solving Eq. (16) for $\frac{\partial q}{\partial S_1}$ and evaluating the result at $S_1 = S$ yields

$$\left.\begin{aligned} &(17) \quad \tan\theta_{gs_1} = \frac{\frac{\partial \beta}{\partial S_1}q^3 + \frac{\partial \gamma}{\partial S_1}q^2 + \frac{\partial \delta}{\partial S_1}q + \frac{\partial \epsilon}{\partial S_1}}{\frac{\partial F(q)}{\partial q}} \\ &(18) \quad \tan\theta_{gS_2} = \frac{\frac{\partial \beta}{\partial S_2}q^3 + \frac{\partial \gamma}{\partial S_2}q^2 + \frac{\partial \delta}{\partial S_2} + \frac{\partial \epsilon}{\partial S_2}}{\frac{\partial F(q)}{\partial q}} \end{aligned}\right\} \text{when } \begin{aligned} S_1 &= S \\ S_2 &= 0 \ . \end{aligned}$$

The derivative of the coefficients of Booker's equation may be obtained from Eqs. (14).

C. Special Cases

The above system of equations gives a rather complete method of determining the ray path. Certain useful cases are now considered to demonstrate the application of these equations. In particular, for $S_1 = S$, $S_2 = 0$, the derivatives of the coefficients of Booker's equation may be obtained in the form

$$\frac{\partial \alpha}{\partial S_1} = 0 , \tag{19a}$$

$$\frac{\partial \beta}{\partial S_1} = 2\ell_1 \ell_3 XY^2 , \tag{19b}$$

$$\frac{\partial \gamma}{\partial S_1} = [4(1-X) - 4Y^2 + 2XY^2(\ell_1{}^2 + \ell_3{}^2)] \quad S , \tag{19c}$$

$$\frac{\partial \delta}{\partial S_1} = - 2\ell_1\ell_3 XY^2(1-3S^2) , \tag{19d}$$

and

$$\frac{\partial \epsilon}{\partial S_1} = [-4(1-X)(C^2-X) + 4C^2Y^2 - 2Y^2X - XY^2\ell_1{}^2(2-4S^2)] \; S, \tag{19e}$$

where $C^2 = 1 - S^2$.

If the static magnetic field $\overline{Y}$ lies in the x-z plane ($\ell_2 = 0$), then the coefficients β, γ, δ, and ϵ depend only on S_1. Thus the partial derivatives with respect to S_2 all vanish for $S_2 = 0$, i.e.,

$$\frac{\partial \alpha}{\partial S_2} = \frac{\partial \beta}{\partial S_2} = \frac{\partial \gamma}{\partial S_2} = \frac{\partial \delta}{\partial S_2} = \frac{\partial \epsilon}{\partial S_2} = 0 \tag{20}$$

and

(21) $$\frac{\partial F(q)}{\partial q} = 4\alpha q^3 + 3\beta q^2 + 2\gamma q + \delta = 0.$$

Several conclusions may be drawn from the above equations. Substituting Eq. (20) into (19) yields

$$\tan \theta_{g_{s_2}} = 0$$

or

$$\theta_{g_{s_2}} = 0.$$

Thus the ray path must lie in the x-y plane when the static magnetic field and the incident ray are in x-y plane, i.e., $S_1 = S$, $S_2 = 0$, and $\ell_2 = 0$.

If the magnetic field is directed along either the x or the z axis, then the four solutions of Booker's equation are related by $q_3 = -q_1$ and $q_4 = -q_2$. These relations also hold for normal incidence, i.e., $S_1 = 0$, without this new restriction on the magnetic field. This last case is of particular interest since the ray path retraces itself upon reflection, in the case of the slab, as illustrated in Fig. 2.

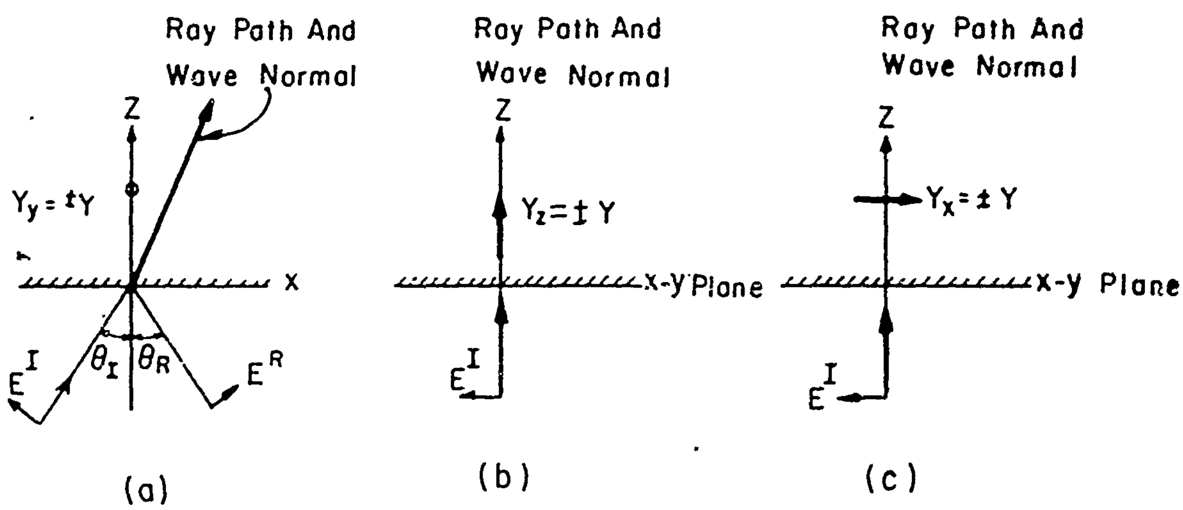

Fig. 2 - Ray path and wave normal of a wave normally incident into an anisotropic medium

It is also of interest to determine the special cases for which the wave normal and the ray path coincide. There are three such cases. If the static magnetic field is oriented in the y direction, i.e., $\ell_2 = 1$ and $\ell_1 = \ell_3 = 0$, then provided $S_1 = S$ and $S_2 = 0$, Eq. (17) becomes

$$\tan \theta_{gs} = \frac{\frac{\partial \gamma}{\partial S} q^2 + \frac{\partial \epsilon}{\partial S}}{4\alpha q^3 + 2\gamma q} .$$

Substituting derivatives obtained from Eq. (14) yields

$$(22) \qquad \tan \theta_{gs} = \frac{[4(1-X)-4Y^2]Sq^2 + [-4(1-X)(C^2-X)+4C^2Y^2-2Y^2X]\ S}{4(1-X-Y^2)q^3 + 2[-2(1-X)(C^2-X)+2Y^2(C^2-X)+XY^2]q}$$
$$= \frac{S}{q} .$$

Equations (3a) and (3b) yield

$$(23) \qquad \tan \theta_p = \frac{S}{q} ,$$

provided $S_2 = 0$, where θ_p is the direction of the wave normal with respect to the z axis. Thus the wave normal and the ray path coincide for this case of the extraordinary wave, which may also be proved by use of the Poynting vector.

If the incident ray is normal to the interface, i.e., $S_1 = S_2 = 0$, and the static magnetic field is oriented in the z direction, i.e., $\ell_3 = 1$ and ℓ_1 and $\ell_2 = 0$, then it can be readily shown from Eqs. (17) and (23) that the ray path and the wave normal again coincide with $\theta_p = \theta_{gs1} = 0$. A similar result is obtained when the static magnetic field is oriented in the x direction. These conclusions are summarized in Fig. 3.

D. Electrical Path Length in the Anisotropic Media

The electrical path length is to be referenced to the point of incidence shown in Fig. 4, which is also the origin of coordinates, i.e., $x = y = z = 0$. Thus the phase of the wave at any point in the medium is given by Eq. (1) as $e^{-jk\phi}$. If the plane of incidence is the x-y plane, i.e., $S_2 = 0$, then

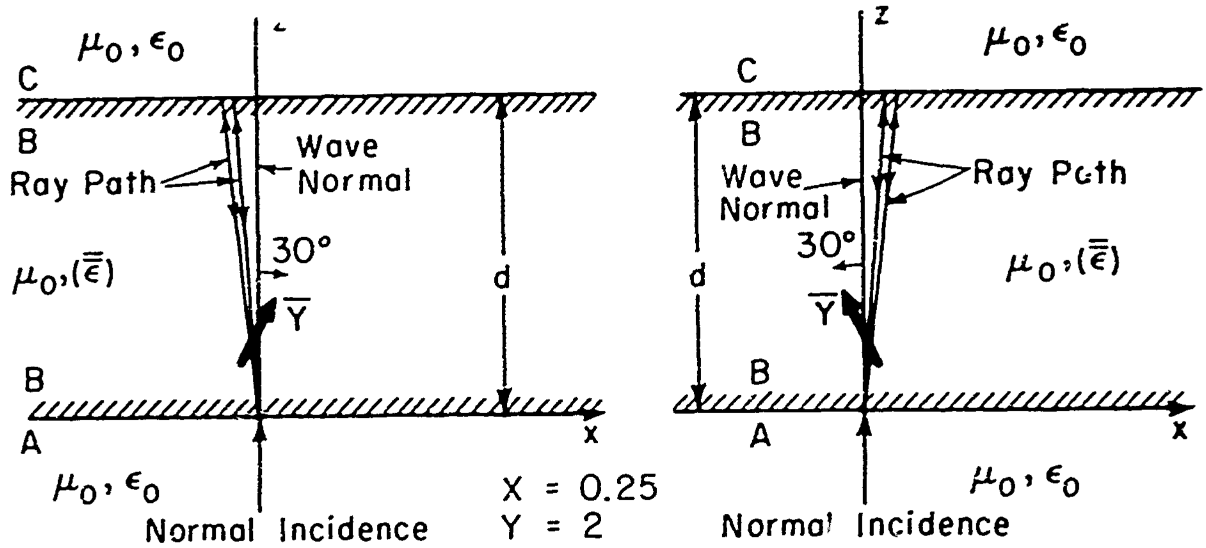

Fig. 3 - Three special cases where the ray path and wave normal coincide

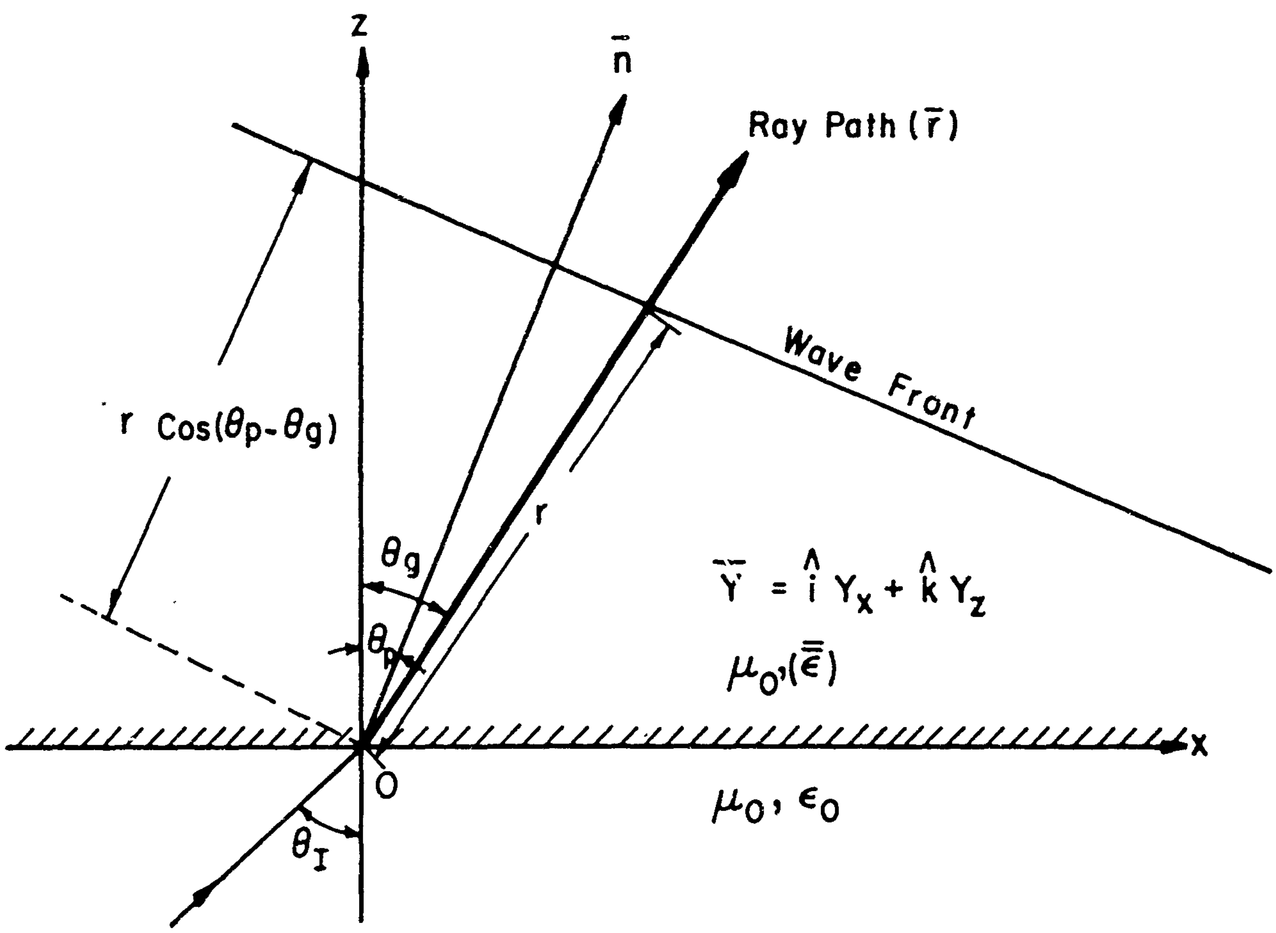

Fig. 4 - Electrical path length in an anisotropic medium

$$\phi = k(Sx + qz) = k(\bar{n} \cdot \bar{r}) . \tag{24}$$

It is seen from Fig. 4 that this can be written in the form

$$\varphi = knr \cos (\theta_g - \theta_p) . \tag{25}$$

It should be noted that this method of obtaining the electrical path length is based on a plane wave incident upon a planar interface. This is generally true in the rigorous application of Snell's law, but its application to other configurations has proved highly accurate and extremely useful. For the present application it is the best available approximation.

E. Modification of S and q on Reflection from a Non-Parallel Interface

All of the equations for the reflection and transmission coefficients at an interface are based on a coordinate system in which the z axis is normal to that interface. If a ray is transmitted through this interface to a second non-parallel interface, as shown in Fig. 5, it becomes necessary to transform our coordinate system in order that previous results be applicable.

Consider the phase of Eq. (24) in the original x, y, z coordinate system. The new coordinates are

$$x' = (x-a) \cos \theta + (z-b) \sin \theta \tag{26}$$

and

$$z' = (z-b) \cos \theta - (x-a) \sin \theta , \tag{27}$$

where z' is normal to the second interface. It follows that

$$x = x' \cos \theta - z' \sin \theta + a \tag{28}$$

and

$$z = z' \cos \theta + x' \sin \theta + b , \tag{29}$$

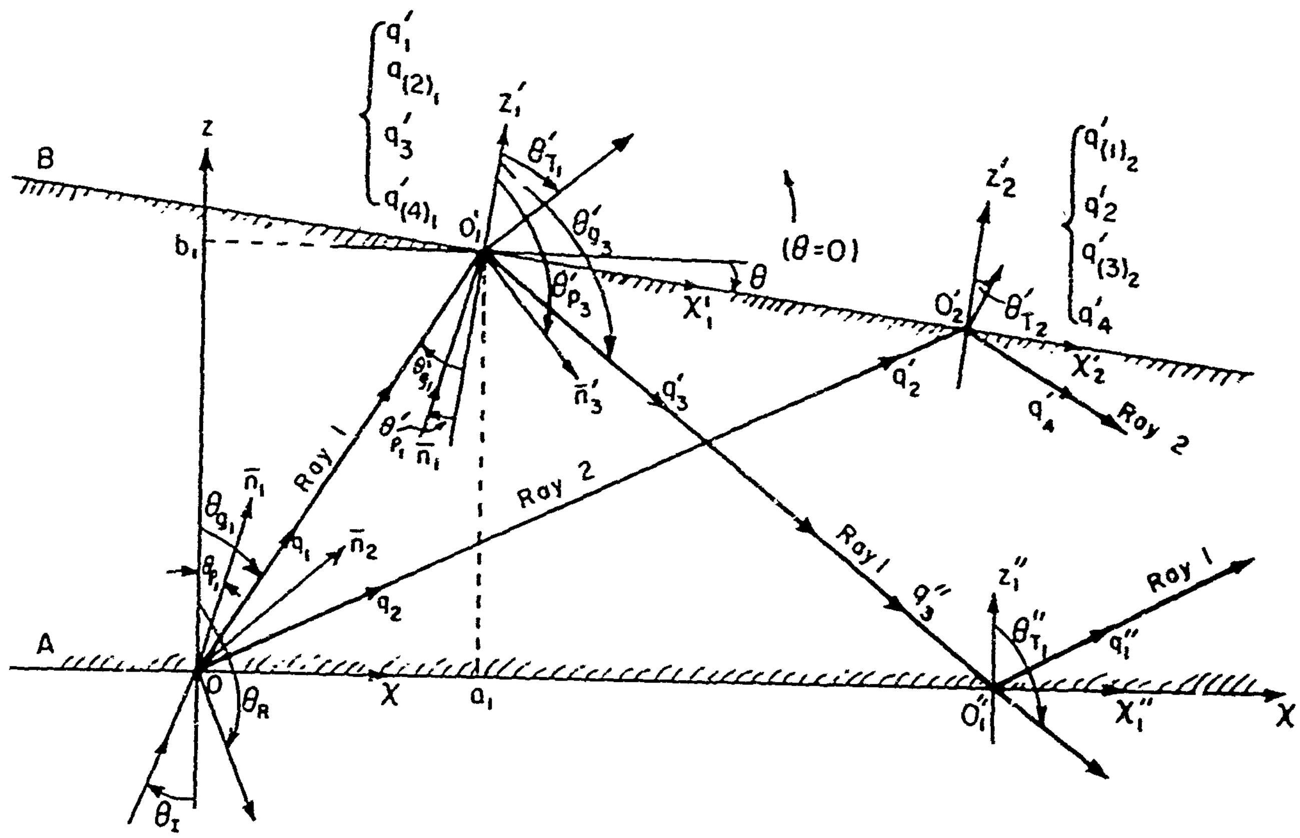

Fig. 5 - Ray tracing for non-parallel interfaces

and Eq. (24) becomes

$$\varphi = kS(x' \cos\theta - z' \sin\theta + a) + kq(z' \cos\theta + x' \sin\theta + b) \tag{30}$$

or

$$\phi = (S\cos\theta + q\sin\theta)\, kx' + (q\cos\theta - S\sin\theta)\, kz' + kSa + kqb. \tag{31}$$

Recalling that

$$S = n \sin\theta_p \; ; \qquad q = n \cos\theta_p , \tag{32}$$

then

(33) $$\phi = kn \sin(\theta_p+\theta)x' + kn \cos(\theta_p+\theta)z' + kn(a \sin \theta_p+b \cos \theta_p),$$

or

(34) $$\varphi = kn \sin(\theta_p+\theta)x' + kn \cos(\theta_p+\theta)z' + k(\bar{n} \cdot \bar{r}) \text{ at } 0' ,$$

or

(35) $$\varphi = kS'x' + kq'z' + k(\bar{n} \cdot \bar{r}) \text{ at } 0' .$$

Note that the x', z' in Fig. 5 for upgoing wave q' are all negative values. In the following, the notation S_1', S_2' indicates the value of S' for ray 1 and ray 2 at boundary B, respectively.

In the transformed coordinate system

(36) $$q' = n \cos(\theta_p + \theta)$$

and

(37) $$S' = n \sin(\theta_p + \theta) .$$

Hence,

(38) $$q' = q \frac{\cos(\theta_p+\theta)}{\cos \theta_p}$$

and

(39) $$S' = S \frac{\sin(\theta_p+\theta)}{\sin \theta_p}$$

As the two interfaces become parallel, ϑ vanishes and S = S' and $S_1' = S_2' = \sin \theta_I$ as before. However, if $\theta \neq 0$, then S' assumes two different values for the two incident waves, as may be seen from Eq. (39), since θ_{p_1}' and θ_{p_2}' differ. Recalling that the coefficients of Booker's quartic equation are dependent on S, it is clear that two different quartic equations must be factored to find the values q_i',

q_2', q_3', and q_4' in the primed system of coordinates. There are now two sets of values for q'. One set of values (q_1', $q'_{(2)_1}$, q_3', $q'_{(4)_1}$) is associated with S_1' at point O_1' (shown in Fig. 5); the other set ($q'_{(1)_2}$, q_2', $q'_{(3)_2}$, q_4') is associated with S_2' at point O_2'. One of the values (q_1' in one set and q_2' in the other) is readily obtained from Eq. (38) for each set. Thus, the two sets of Booker's quartic equations now may be reduced to two sets of cubic equations, which must be factored to find the proper values of q_3' from one set associated with q_1' and q_4' from the other set associated with q_2'.

In other words, ray tracing is accomplished by means of the equation

$$\alpha' q'^4 + \beta' q'^3 + \gamma' q'^2 + \delta' q' + \epsilon' = 0 \tag{40}$$

for the prime system, where β', γ', δ', and ϵ' are functions of S' and $\frac{\partial \beta'}{\partial S'}, \frac{\partial \gamma'}{\partial S'}, \frac{\partial \delta'}{\partial S'}, \frac{\partial \epsilon'}{\partial S'}$ are all known quantities. For ray 1 at point O_1' (see Fig. 5), since q_1' is known, Eq. (40) reduces to

$$\alpha_1' q'^3 + (\beta_1' + \alpha_1' q_1')q'^2 + (\gamma_1' + \beta_1' q_1' + \alpha_1' q_1'^2)\, q' + (\delta_1' + \gamma_1' q_1' + \beta_1' q_1'^2 + \alpha_1' q_1'^3) = 0, \tag{41}$$

where α_1', β_1', γ_1', and δ_1' are associated with S_1'. Once the three solutions, $q'_{(2)_1}$, q_3', $q'_{(4)_1}$, are obtained from Eq. (41), the relation has been established that if

$$q_1' > q'_{(2)_1} \quad \text{then} \tag{42}$$

$$q_3' < q'_{(4)_1},$$

and q_3' is the solution we are seeking for ray 1, on the assumption that ${}_1R_4$ and ${}_2R_3$ are negligible. Once q_3' is obtained, the angle θ'_{p_3} is given by

$$\tan \theta'_{p_3} = \frac{S_1'}{q_3'}, \tag{43}$$

and θ'_{g_3} is obtained from Eq. (17).

Similarly, for ray 2 at point O_2' we may obtain q_4' from a cubic equation similar to Eq. (41), but with respect to S_2' and q_2' at point O_2'. The coefficients of the Booker's equation may be replaced by α_2', β_2', γ_2', and δ_2' in Eq. (41). The q_4' is chosen by the relation as shown in Eq. (42) (if $q_2' < q'_{(1)_2}$, then $q_4' > q'_{(3)_2}$). The technique applied for ray 2 is the same as for ray 1. However $q'_{(4)_1}$ is needed to calculate ${}_1'R_3'$, and $q'_{(3)_2}$ is needed to calculate ${}_2'R_4'$.

We need to note that the symbols S_1', S_2' used here should not be confused with the symbols S_1, S_2 used in Eqs. (17) and (18). We have mentioned that the plane of the incident wave is the x-z plane, hence we use S instead of S_1; and S_2, which is zero, will not appear in the following. When the incident wave reaches the boundary A (shown in Fig. 5), S is equal to $\sin \theta_I$. Once the incident wave is traveling in the anisotropic medium, it will split into two waves. One of them reaches boundary B at O_1' (shown in Fig. 5) as ray 1. The other reaches boundary B at O_2' as ray 2.

At boundary B, the prime coordinate system is used; hence,

$$S_1' = \sin \theta'_{T_1} \qquad \text{(for ray 1)}$$

and

$$S_2' = \sin \theta'_{T_2} \qquad \text{(for ray 2)},$$

where θ'_{T_1}, θ'_{T_2} are shown in Fig. 5.

At this point, the effect of the skewed interface has been completely determined.

III. SUMMARY OF RAY OPTICS FOR ARBITRARY GYROTROPIC BODIES

All of the fundamentals needed to trace a ray path through a finite, homogeneous, gyrotropic body, and to compute the magnitude and phase of the emergent rays, have now been developed by considering only planar interfaces. This might appear to be a severe restriction; however, modified geometrical optics as applied to a general isotropic body is also based on the planar interfaces. Snell's

law and the various reflection and transmission coefficients are all derived under the assumption of infinite planar interfaces. Yet this modified geometrical optics method has been remarkably successful in computing the radar cross section for isotropic spheres and cylinders. This modified geometrical optics method also yields remarkably accurate radar cross sections for the plasma cylinder with an axial static magnetic field[11].

The same assumption made in developing the modified geometrical optics method in these previous cases would also be made here for this general anisotropic case. That assumption is that the planar interfaces of Fig. 5 would represent the tangent planes of any curved body at the point of incidence and the point of reflection. The ray technique which has been developed for finite homogeneous anisotropic bodies will now be summarized.

A ray is incident at the origin as shown in Fig. 5. The value of S is obtained from the incident angle θ_I as $S = \sin \theta_I$ and $C = \cos \theta_I$, where it is recalled that the incident plane wave is given by

$$U^I = U_o \, e^{-jk(Sx+Cz)} .$$

The parameters of the gyrotropic plasma medium are given by X and $\overline{Y}$ which are given in Appendix II. The direction cosines, ℓ_1 and ℓ_3, of the magnetic field vector are obtained in the x, y, z coordinate system. These represent the physical parameters of the anisotropic body to be treated. If that body has a curved surface it should be noted that S is a function of the point at which the ray enters the body for a particular plane wave incidence; or, conversely, that the coordinate system of Fig. 5 is always chosen so that the z axis is normal to the surface at the point of incidence which coincides with the origin. Thus the values of S, C, ℓ_1, ℓ_2, and ℓ_3 are all functions of this angle of incidence θ_I.

Once θ_I is found, values of S, C, ℓ_1, ℓ_2, and ℓ_3 are readily obtained. These parameters uniquely determine the values of the coefficients of Booker's quartic equation ($\alpha, \beta, \gamma, \delta, \epsilon$) given by Eqs. (14a-e).

It should be recalled that for our present case ℓ_2 is set equal to zero to maintain the refracted angle in the plane of incidence. This is not an essential step but simply reduces the complexity of the solution. The more general case of $\ell_2 \neq 0$ would follow the procedures given

without any complication.

Once α, β, γ, δ, and ϵ are obtained, Booker's quartic equation, Eq. (14), may be factored and the four values of q obtained.

The two roots q_1, q_2 (for which the signs of the ray path angles θ_{g1} and θ_{g_2} are the same as θ_I) are the ones being sought since they represent waves traveling in the positive z direction. Values of q_1, q_2 may not always be positive and represent waves traveling in positive z direction. However, the ray follows the path along which the energy flows. This is a practical way to check whether the q_1, q_2 are correct.

The values of the angles θ_{g_1} and θ_{g_2} are obtained from Eq. (17). These angles are the directions of the two ray paths associated with q_1 and q_2, respectively.

The angles θ_{p_1} and θ_{p_2} of the refractive index n will be useful in the following calculations, and are readily obtained from Eq. (23) using values of q_1 and q_2, respectively. The values of n_1 and n_2 are obtained from Eq. (3). The relative phase of the ray at the point of intersection with the second interface is obtained by use of Eq. (25). It is necessary to determine this relative phase since the phasor sum of the various scattered field components is to be computed to find the total scattered field.

In order to specify the magnitude of the ray at the point of intersection on the second interface, it is necessary to compute the transmission coefficients at the first interface. These are given by Eq. (49). At this time the reflection coefficient, which would be associated with the direct reflected ray, may also be computed at the first interface.

The fields associated with the two rays at their point of intersection with the second interface are given by

$$U^T_{(1A)} = U_{o\,\|}T_1 \, e^{-jk[n_1 r_1 \cos(\theta_{p_1} - \theta_{g_1})]}$$

and

$$U^T_{(2A)} = U_{o\,\|}T_2 \, e^{-jk[n_2 r_2 \cos(\theta_{p_2} - \theta_{g_2})]} ,$$

where r_1 and r_2 are the lengths of the ray paths. The direct reflected fields are given by

$$_{\parallel}U^R_{\parallel(A)} = U_o \, _{\parallel}R_{\parallel o} \, e^{-jk(Sx-Cz)}$$

and

$$_{\parallel}U^R_{\perp(A)} = U_o \, _{\parallel}R_{\perp o} \, e^{-jk(Sx-Cz)} \quad .$$

It is now necessary to determine a new set of parameters associated with the transmission and reflection at this second interface. The first step is to transform the coordinate system such that the z_1' axis is now normal to the second interface, as illustrated by the x_1', y_1', z_1' coordinate system for ray 1 shown in Fig. 5. The values q_1' and S_1' of the ray incident on this second interface in this primed coordinate system are given by Eqs. (38) and (39). The subscript (1) on the coordinate axes refers to the system associated with q_1'. A second coordinate system designated by x_2', y_2', z_2' is associated with q_2' for ray 2. This is necessary because ray paths r_1 and r_2 differ.

The value $C_1'^2 = 1 - S_1'^2$ and the values of the direction cosines ℓ_1' and ℓ_3' of the magnetic field are readily obtained in this new coordinate system. The coefficients of Booker's quartic equation (α', β', γ', δ', ϵ') now may be obtained from Eqs. (14a-e).

For ray 1, these coefficients are functions of S_1' at point O_1'; thus Booker's equation must be factored to obtain the roots q_1' $q'_{(2)1}$, q_3', $q'_{(4)1}$. However q_1' is already known and Booker's equation reduces to a cubic equation: Eq. (41). This equation is factored to obtain the remaining three roots $q'_{(2)1}$, q_3', $q'_{(4)1}$. The coupling term is to be neglected in the following work but there is no real need to do so except to minimize complications.

The desired value of q'_3 is chosen so that if $q'_1 > q'_{(2)1}$, then $q'_3 < q'_{(4)1}$, and the associated pair (q'_1, q'_3) is being sought. Now (for ray 1) θ'_{g_3}, as shown in Fig. 5, is found from Eq. (17), and θ'_{p_3} from Eq. (43). The index of refraction, n'_3, is found from Eq. (32). The same technique can be applied to find q'_4 from q'_2 at point O_2 for ray 2.

The point of intersection of these ray paths with the next interface may then be found for the geometry being treated. In the example of

Fig. 5, this is the original interface. Calculation of phase at this intersection follows the method described above. The reflection and transmission coefficients at this boundary, i.e., at the point O' of Fig. 5, are given by Eq. (50). At point O'_1 the values q'_1, q'_3, $q'_{(4)1}$ are used to calculate ${}_1'R_3'$ and ${}_1'T_{(\parallel \text{ or } \perp)}$; at point O_2 the values q'_2, q'_4, $q'_{(3)2}$ are used to calculate ${}_2'R_4'$ and ${}_2'T_{(\parallel \text{ or } \perp)}$.

The fields transmitted into free space at the second interface are given by

$$ {}_{\parallel}U^T_{\parallel(1B)} = U_o \, {}_{\parallel}T_1 \, {}_1'T_{\parallel} \, e^{-j\phi_1} \, e^{-jk(S_1'x_1' + C_1'z_1')}, $$

$$ {}_{\parallel}U^T_{\perp(1B)} = U_o \, {}_{\parallel}T_1 \, {}_1'T_{\perp} \, e^{-j\phi_1} \, e^{-jk(S_1'x_1' + C_1'z_1')}, $$

$$ {}_{\parallel}U^T_{\parallel(2B)} = U_o \, {}_{\parallel}T_2 \, {}_2'T_{\parallel} \, e^{-j\phi_2} \, e^{-jk(S_2'x_2' + C_2'z_2')}, $$

and

$$ {}_{\parallel}U^T_{\perp(2B)} = U_o \, {}_{\perp}T_2 \, {}_2'T_{\perp} \, e^{-j\phi_2} \, e^{-jk(S_2'x_2' + C_2'z_2')} ; $$

where

$$ \phi_1 = kn_1 r_1 \cos(\theta_{p_1} - \theta_{g_1}) $$

$$ \phi_2 = kn_2 r_2 \cos(\theta_{p_2} - \theta_{g_2}) $$

and all phases are referenced to the original origin of coordinates at point O, (i.e., $x_1 = y = z = 0$). The fields of the ray reflected back to the anisotropic medium to the point O_1'' and O_2'' are

$$ U^R_{(1B)} = U_o \, {}_{\perp}T_1 \, {}_1R_3 \, e^{-j(\phi_1 + \phi_1')} $$

and

$$ U^R_{(2B)} = U_o \, {}_{\parallel}T_2 \, {}_2'R_4' \, e^{-j(\phi_2 + \phi_2')} , $$

where

$$\varphi_1' = kn_3' r_3' \cos(\theta_{p_3}' - \theta_{g_3}')$$

and

$$\varphi_2' = kn_4' r_4' \cos(\theta_{p_4}' - \theta_{p_4}') ,$$

and all phases are referenced to the original origin O.

By the use of ray optics for the anisotropic body, a ray incident upon the body from the external medium has been considered and followed completely through one internal reflection. All of the fields associated with this case have been given; and any additional internal reflections may be treated simply by repeating these same steps. In addition any coupling terms may also be readily included in any case where coupling becomes significant.

Any changes in amplitude introduced by diverging ray systems, i.e., spatial attenuation, have been neglected. However, this problem can be handled by the modified geometrical optics method.

APPENDIX I
BOOKER'S QUARTIC EQUATION

The wave form is

$$\mathrm{EXP}\{-jk(S_1 x + S_2 y + qz)\} \,,$$

and we may write symbolically

$$\frac{\partial}{\partial x} = -jkS_1 \,,$$

$$\frac{\partial}{\partial y} = -jkS_2 \,,$$

and

$$\frac{\partial}{\partial z} = -jkq,$$

where Maxwell's equation give

$$(44) \qquad \begin{bmatrix} 0 & -q & S_2 \\ q & 0 & -S_1 \\ -S_2 & S_1 & 0 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \eta_o \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix}$$

$$\eta_o \begin{bmatrix} 0 & -q & S_2 \\ q & 0 & -S_1 \\ -S_2 & S_1 & 0 \end{bmatrix} \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} = \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} - \frac{1}{\epsilon_o} \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} \,.$$

Combining the above equations and using the relation

$$\frac{1}{\epsilon_o} \, [P] = [M] \, [E] \,,$$

we get

(45)
$$\begin{bmatrix} 1-q^2-S_2^2+M_{xx} & S_1S_2+M_{xy} & S_1q+M_{xz} \\ S_1S_2+M_{yx} & 1-q^2-S_1^2+M_{yy} & S_2q+M_{yz} \\ S_1q+M_{zx} & S_2q+M_{zy} & C^2+M_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0 .$$

The form of the above equation becomes

(46)
$$F(q) = \alpha q^4 + \beta q^3 + \gamma q^2 + \delta q + \epsilon = 0 ,$$

which is the well-known Booker's quartic equation.

Using the elements of [M] we obtain

$$\alpha = U(U^2 - Y^2) + X(Y_z^2 - U^2) ,$$

$$\beta = 2X \ (S_1 Y_z Y_x + S_2 Y_z Y_y) ,$$

$$\gamma = -2U(U-X)(C^2U-X) + 2Y^2(C^2U-X) +$$

$$X \ \{Y^2 - C^2 Y_z^2 + (S_1 Y_x + S_2 Y_y)^2\} ,$$

$$\delta = -2C^2X \ (S_1 Y_z Y_x + Y_z Y_y) ,$$

and

$$\epsilon = (U-X)(C^2U-X)^2 - C^2Y^2(C^2U-X) - C^2X \ (S_1 Y_x + S_2 Y_y)^2 .$$

All symbols X, Y, U are defined in Appendix II.

APPENDIX II
THE SUSCEPTIBILITY MATRIX

Let

$$X = \frac{\omega_N^2}{\omega^2} = \frac{Ne^2}{\epsilon_o m \omega^2} ,$$

$$\overline{Y} = \frac{\overline{\omega}_H}{\omega} = \frac{\mu_o e \overline{H}_o}{m\omega} = Y_x \hat{i}_x + Y_y \hat{i}_y + Y_z \hat{i}_z ,$$

$$U = 1 - jZ = 1 - j\frac{\nu}{\omega} ,$$

ω = operation frequency,

ν = collision frequency,

ω_H = cyclotron frequency,

ω_N = plasma frequency,

m = mass of electrons, and

ϵ_o = permittivity in free space,

μ_o = permeability in free space,

N = number of electrons per unit volume,

e = charge of an electron,

H_o = applied static magnetic field.

The constitutive relation

$$-\epsilon_o X \overline{E} = U\overline{P} + j(\overline{Y} \times \overline{P})$$

or

$$\epsilon_o[E] = [Y][P] ,$$

where

(47)
$$[Y] = -\frac{1}{X}\begin{bmatrix} U & -jY_z & jY_y \\ jY_z & U & -jY_x \\ -jY_y & jY_x & U \end{bmatrix},$$

gives

$$\frac{1}{\epsilon_o}[P] = [Y]^{-1}[E] = [M][E],$$

where

(48)
$$[M] = \frac{-X}{U(U^2-Y^2)}\begin{bmatrix} U^2-Y_x & jUY_z-Y_xY_y & -jUY_y-Y_xY_z \\ -jUY_z-Y_xY_y & U^2-Y_y^2 & jUY_x-Y_yY_z \\ jUY_y-Y_xY_z & -jUY_x-Y_yY_z & U^2-Y_z^2 \end{bmatrix}$$

and $\overline{P}$ is electric polarization.

APPENDIX III
REFLECTION AND TRANSMISSION COEFFICIENTS AT PLANAR "FREE-SPACE GYROTROPIC" INTERFACES

The coordinate system used in this Appendix is chosen so that $S_1 = S$ and $S_2 = 0$. There are four possible waves in the gyroptropic medium; paired as upgoing waves, designated by subscripts (1) and (2), and downgoing waves, designated by subscripts (3) and (4). Thus a wave (1) or (2) in the gyrotropic medium may be reflected as wave (3) or (4), the reflection coefficient of which is designated ${}_1R_3$, ${}_1R_4$, ${}_2R_3$, or ${}_2R_4$, etc. The quantities η, π are obtained from Eq. (9) by setting $S_2 = 0$.

A. Wave Incident From Free Space up to Gyrotropic Medium

For parallel polarized plane wave incidence ($E_y^I = 0$)

(49a) $$_{\parallel}T_1 = \frac{E_{z1}^T}{\eta H_y^I} = C\,\frac{E_{z1}^T}{E_x^I} = \frac{2C}{D_1}\left(\pi_{y_2} - \frac{1}{C}\eta_{x_2}\right),$$

$$_{\parallel}T_2 = \frac{E_{z_2}^I}{\eta H_y^I} = C\,\frac{E_{z_2}^T}{E_x^I} = -\frac{2C}{D_1}\left(\pi_{y_1} - \frac{1}{C}\eta_{x_1}\right),$$

$$_{\parallel}R_{\parallel} = \frac{H_y^R}{H_y^I} = \frac{E_x^R}{E_x^I} = \frac{1}{D_1}\Big[(\eta_{x_1}\eta_{y_2} - \eta_{y_1}\eta_{x_2}) + \frac{1}{C}(\pi_{x_1}\eta_{x_2} - \eta_{x_1}\pi_{x_2})$$
$$- C(\pi_{y_1}\eta_{y_2} - \eta_{y_1}\pi_{y_2}) - (\pi_{x_1}\pi_{y_2} - \pi_{y_1}\pi_{x_2})\Big],$$

and

$$_{\parallel}R_{\perp} = \frac{E_y^R}{\eta_o H_y^I} = -\frac{2}{D_1}\left[\pi_{y_1}\eta_{x_2} - \eta_{x_1}\pi_{y_2}\right].$$

For perpendicular polarized plane wave incidence ($E_x^I = 0$),

(49b) $$_{\perp}T_1 = \frac{E_{z_1}^T}{E_y^I} = -\frac{2}{D_1}\left(\pi_{x_2} - C\eta_{y_2}\right).$$

(49b) (cont) $${}_{\perp}T_2 = \frac{E_{z_2}{}^T}{E_y{}^I} = \frac{2}{D_1}(\pi_{x_1} + C\eta_{y_1}),$$

$${}_{\perp}R_{\perp} = \frac{E_y{}^R}{E_y{}^I} = \frac{1}{D_1}\Big[-(\eta_{x_1}\eta_{y_2} - \eta_{y_1}\eta_{x_2}) + \frac{1}{C}(\pi_{x_1}\eta_{x_2} - \eta_{x_1}\pi_{x_2})$$

$$- C(\pi_{y_1}\eta_{y_2} - \eta_{y_1}\pi_{y_2}) + (\pi_{x_1}\pi_{y_2} - \pi_{y_1}\pi_{x_2})\Big],$$

and

$${}_{\perp}R_{\parallel} = \frac{\eta_o H_y{}^R}{E_y{}^I} = \frac{2}{D_1}(\pi_{x_1}\eta_{y_2} - \eta_{y_1}\pi_{x_2}),$$

where

$$D_1 = (\eta_{x_1}\eta_{y_2} - \eta_{y_1}\eta_{x_2}) - \frac{1}{C}(\pi_{x_1}\eta_{x_2} - \eta_{x_1}\pi_{x_2})$$

$$- C(\pi_{y_1}\eta_{y_2} - \eta_{y_1}\pi_{y_2}) + (\pi_{x_1}\pi_{y_2} - \pi_{y_1}\pi_{x_2}).$$

Transmission into the gyrotropic medium yields coefficients of the form ${}_{\parallel}T_1$, ${}_{\parallel}T_2$, where the subscript $\parallel$ designates the polarization of the incident wave and (1) or (2) designates one of the upgoing waves. Conventional definitions of polarization are used so that subscript $\parallel$ or $\perp$ means that the E vector is "parallel to" or is "perpendicular to" the plane of incidence, respectively.

B. Wave Incident From Gyrotropic Medium Up to Free Space

For wave (1) ($E_{z_2}{}^I = 0$)

(50a) $${}_{1}T_{\parallel} = \frac{\eta H_y{}^T}{E_{z_1}{}^I} = \frac{1}{C}\frac{E_x{}^T}{E_{z_1}{}^I} = \frac{\frac{1}{C}}{D_2}\{\pi_{x_1}(\eta_{x_3}\eta_{y_4} - \eta_{y_3}\eta_{x_4}) - \eta_{x_1}(\pi_{x_3}\eta_{y_4} - \eta_{y_3}\pi_{x_4})$$

$$+ \eta_{y_1}(\pi_{x_3}\eta_{x_4} - \eta_{x_3}\pi_{x_4}) + C\pi_{x_1}(\pi_{y_3}\eta_{y_4} - \eta_{y_3}\pi_{y_4})$$

$$- C\pi_{y_1}(\pi_{x_3}\eta_{y_4} - \eta_{y_3}\pi_{x_4}) + C\eta_{y_1}(\pi_{x_3}\pi_{y_4} - \pi_{y_3}\pi_{x_4})\},$$

(50a) (cont)

$$ {}_1T_{\perp} = \frac{E_y{}^T}{E_{z1}{}^I} = \frac{1}{D_2}\{\pi_{y1}(\eta_{x_3}\eta_{y_4}-\eta_{y_3}\eta_{x_4}) - \eta_{x1}(\pi_{y_3}\eta_{y_4}-\eta_{y_3}\pi_{y_4}) $$

$$ + \eta_{y1}(\pi_{y_3}\eta_{x_4}-\eta_{x_3}\pi_{y_4}) - \frac{1}{C}\pi_{x1}(\pi_{y_3}\eta_{x_4}-\eta_{x_3}\pi_{y_4}) $$

$$ + \frac{1}{C}\pi_{y1}(\pi_{x_3}\eta_{x_4}-\eta_{x_3}\pi_{x_4}) - \frac{1}{C}\eta_{x1}(\pi_{x_3}\pi_{y_4}-\pi_{y_3}\pi_{x_4})\}, $$

$$ {}_1R_3 = \frac{E_{z_3}{}^R}{E_{z1}{}^I} = -\frac{1}{D_2}[(\eta_{x1}\eta_{y_4}-\eta_{y1}\eta_{x_4}) + \frac{1}{C}(\pi_{x1}\eta_{x_4}-\eta_{x1}\pi_{x_4}) $$

$$ + C(\pi_{y_1}\eta_{y_4}-\eta_{y1}\pi_{y_4}) + (\pi_{x1}\pi_{y_4}-\pi_{y1}\pi_{x_4})], $$

and

$$ {}_1R_4 = \frac{E_{z_4}{}^R}{E_{z_1}{}^I} = \frac{1}{D_2}[(\eta_{x1}\eta_{y_3}-\eta_{y1}\eta_{x_3}) + \frac{1}{C}(\pi_{x1}\eta_{x_3}-\eta_{x1}\pi_{x_3}) $$

$$ + C(\pi_{y_1}\eta_{y_3}-\eta_{y1}\pi_{y_3}) + (\pi_{x1}\pi_{y_3}-\pi_{y1}\pi_{x_3})]. $$

For wave (2) $(E_{z_1}{}^I = 0)$

(50b)

$$ {}_2T_{\parallel} = \frac{\eta H_y{}^T}{E_{z_2}{}^I} = \frac{1}{C}\frac{E_x{}^T}{E_{z_2}{}^I} = \frac{\frac{1}{C}}{D_2}\{\pi_{x_2}(\eta_{x_3}\eta_{y_4}-\eta_{y_3}\eta_{x_4}) - \eta_{x_2}(\pi_{x_3}\eta_{y_4}-\eta_{y_3}\pi_{x_4}) $$

$$ + \eta_{y_2}(\pi_{x_3}\eta_{x_4}-\eta_{x_3}\pi_{x_4}) + C\pi_{x_2}(\pi_{y_3}\eta_{y_4}-\eta_{y_3}\pi_{y_4}) $$

$$ - C\pi_{y_2}(\pi_{x_3}\eta_{y_4}-\eta_{y_3}\pi_{x_4}) + C\eta_{y_2}(\pi_{x_3}\pi_{y_4}-\pi_{y_3}\pi_{x_4})\}, $$

$$ {}_2T_{\parallel} = \frac{E_y{}^T}{E_{z_2}{}^I} = \frac{1}{D_2}\{\pi_{x_2}(\eta_{x_3}\eta_{y_4}-\eta_{y_3}\eta_{x_4}) - \eta_{x_2}(\pi_{y_3}\eta_{y_4}-\eta_{y_3}\pi_{y_4}) $$

$$ + \eta_{y_2}(\pi_{y_3}\eta_{x_4}-\eta_{x_3}\pi_{y_4}) - \frac{1}{C}\pi_{x_2}(\pi_{y_3}\eta_{x_4}-\eta_{x_3}\pi_{y_4}) $$

$$ + \frac{1}{C}\pi_{y_2}(\pi_{x_3}\eta_{x_4}-\eta_{x_3}\pi_{x_4}) - \frac{1}{C}\eta_{x_2}(\pi_{x_3}\pi_{y_4}-\pi_{y_3}\pi_{x_4})\}, $$

$$ (50b) \text{ (cont)} \quad {}_2R_3 = \frac{-1}{D_2}\left[(\eta_{x_2}\eta_{y_4}-\eta_{y_2}\eta_{x_4}) + \frac{1}{C}(\pi_{x_2}\eta_{x_4}-\eta_{x_2}\pi_{x_4}) + C(\pi_{y_2}\eta_{y_4}-\eta_{y_2}\pi_{y_4}) + (\pi_{x_2}\pi_{y_4}-\pi_{y_2}\pi_{x_4})\right], $$

and

$$ {}_2R_4 = \frac{E_{z_4}{}^R}{E_{z_2}{}^I} = \frac{1}{D_2}\left[(\eta_{x_2}\eta_{y_3}-\eta_{y_2}\eta_{x_3}) + \frac{1}{C}(\pi_{x_2}\eta_{x_3}-\eta_{x_2}\pi_{x_3}) + C(\pi_{y_2}\eta_{y_3}-\eta_{y_2}\pi_{y_3}) + (\pi_{x_2}\pi_{y_3}-\pi_{y_2}\pi_{x_3})\right], $$

where

$$ D_2 = (\eta_{x_3}\eta_{y_4}-\eta_{y_3}\eta_{x_4}) + \frac{1}{C}(\pi_{x_3}\eta_{x_4}-\eta_{x_3}\pi_{x_4}) + C(\pi_{y_3}\eta_{y_4}-\eta_{y_3}\pi_{y_4}) + (\pi_{x_3}\pi_{y_4}-\pi_{y_3}\pi_{x_4}) . $$

The dominant reflection coefficients are ${}_1R_3$ and ${}_2R_4$. Coefficients ${}_1R_4$ and ${}_2R_3$ are usually negligible.

C. Wave Incident From Gyrotropic Medium Down to Free Space

For wave (3) $(E_{z_4}{}^I = 0)$

$$ (51a) \quad {}_3T_{\parallel} = \frac{\eta H_y{}^T}{E_{z_3}{}^I} = -C\frac{E_x{}^T}{E_{z_3}{}^I} = \frac{-C}{D_3}\{\pi_{x_3}(\eta_{x_1}\eta_{y_2}-\eta_{y_1}\eta_{x_2}) - \eta_{x_3}(\pi_{x_1}\eta_{y_2}-\eta_{y_1}\pi_{x_2}) + \eta_{y_3}(\pi_{x_1}\eta_{x_2}-\eta_{x_1}\pi_{x_2}) - C\,\pi_{x_3}(\pi_{y_1}\eta_{y_2}-\eta_{y_1}\pi_{y_2}) + C\pi_{y_3}(\pi_{x_1}\eta_{y_2}-\eta_{y_1}\pi_{x_2}) - C\eta_{y_3}(\pi_{x_1}\pi_{y_2}-\pi_{y_1}\pi_{x_2})\}, $$

(51a) (cont)

$$ {}_3T_{\perp} = \frac{E_y{}^T}{E_{z_3}{}^I} = \frac{1}{D_3}\{\pi_{y_3}(\eta_{x_1}\eta_{y_2}-\eta_{y_1}\eta_{x_2}) - \eta_{x_3}(\pi_{y_1}\eta_{y_2}-\eta_{y_1}\pi_{y_2}) $$

$$ + \eta_{y_3}(\pi_{y_1}\eta_{x_2}-\eta_{x_1}\pi_{y_2}) + \frac{1}{C}\pi_{x_3}(\pi_{y_1}\eta_{x_2}-\eta_{x_1}\pi_{y_2}) $$

$$ - \frac{1}{C}\pi_{y_3}(\pi_{x_1}\eta_{x_2}-\eta_{x_1}\pi_{x_2}) + \frac{1}{C}\eta_{x_3}(\pi_{x_1}\pi_{y_2}-\pi_{y_1}\pi_{x_2})\}, $$

$$ {}_3R_1 = \frac{E_{z_1}{}^R}{E_{z_3}{}^I} = \frac{1}{D_3}\{(\eta_{x_2}\eta_{y_3}-\eta_{y_2}\eta_{x_3}) - \frac{1}{C}(\pi_{x_2}\eta_{x_3}-\eta_{x_2}\pi_{x_3}) $$

$$ - C(\pi_{y_2}\eta_{y_3}-\eta_{y_2}\pi_{y_3}) + (\pi_{x_2}\pi_{y_3}-\pi_{y_2}\pi_{x_3})\}, $$

and

$$ {}_3R_2 = \frac{E_{z_2}{}^R}{E_{z_3}{}^I} = \frac{1}{D_3}\{-(\eta_{x_1}\eta_{y_3}-\eta_{y_1}\eta_{x_3}) + \frac{1}{C}(\pi_{x_1}\eta_{x_3}-\eta_{x_1}\pi_{x_3}) $$

$$ + C(\pi_{y_1}\eta_{y_3}-\eta_{y_1}\pi_{y_3}) - (\pi_{x_1}\pi_{y_3}-\pi_{y_1}\pi_{x_3})\}. $$

For wave (4) $(E_{z_3}{}^I = 0)$

(51b)

$$ {}_4T_{\parallel} = \frac{\eta H_y{}^T}{E_{z_4}{}^I} = -C\frac{E_x{}^T}{E_{z_4}{}^I} = -\frac{C}{D_3}\{\pi_{x_4}(\eta_{x_1}\eta_{y_2}-\eta_{y_1}\eta_{x_2}) $$

$$ -\eta_{x_4}(\pi_{x_1}\eta_{y_2}-\eta_{y_1}\pi_{x_2}) + \eta_{y_4}(\pi_{x_1}\eta_{x_2}-\eta_{x_1}\pi_{x_2}) $$

$$ - C\pi_{x_4}(\pi_{y_1}\eta_{y_2}-\eta_{y_1}\pi_{y_2}) + C\pi_{y_4}(\pi_{x_1}\eta_{y_2}-\eta_{y_1}\pi_{x_2}) $$

$$ - C\eta_{y_4}(\pi_{x_1}\pi_{y_2}-\pi_{y_1}\pi_{x_2})\}, $$

$$ {}_4T_{\perp} = \frac{E_y{}^T}{E_{z_3}{}^I} = \frac{1}{D_3}\{\pi_{y_4}(\eta_{x_1}\eta_{y_2}-\eta_{y_1}\eta_{x_2}) - \eta_{x_4}(\pi_{y_1}\eta_{y_2}-\eta_{y_1}\pi_{y_2}) $$

$$ + \eta_{y_4}(\pi_{y_1}\eta_{x_2}-\eta_{x_1}\pi_{y_2}) + \frac{1}{C}\pi_{x_4}(\pi_{y_1}\eta_{x_2}-\eta_{x_1}\pi_{y_2}) $$

(51b)
(cont)

$$- \frac{1}{C} \pi_{y_4}(\pi_{x_1}\eta_{x_2} - \eta_{x_1}\pi_{x_2}) + \frac{1}{C}\eta_{x_4}(\pi_{x_1}\pi_{y_2} - \pi_{y_1}\pi_{x_2})\},$$

$$_4R_1 = \frac{E_{z_1}{}^R}{E_{z_4}{}^I} = \frac{1}{D_3}\{(\eta_{x_2}\eta_{y_4} - \eta_{y_2}\eta_{x_4}) - \frac{1}{C}(\pi_{x_2}\eta_{x_4} - \eta_{x_2}\pi_{x_4})$$

$$- C(\pi_{y_2}\eta_{y_4} - \eta_{y_2}\pi_{y_4}) + (\pi_{x_2}\pi_{y_4} - \pi_{y_2}\pi_{x_4})\},$$

and

$$_4R_2 = \frac{E_{z_2}{}^R}{E_{z_4}{}^I} = \frac{1}{D_3}\{-(\eta_{x_1}\eta_{y_4} - \eta_{y_1}\eta_{x_4}) + \frac{1}{C}(\pi_{x_1}\eta_{x_4} - \eta_{x_1}\pi_{x_4})$$

$$+ C(\pi_{y_1}\eta_{y_4} - \eta_{y_1}\pi_{y_4}) - (\pi_{x_1}\pi_{y_4} - \pi_{y_1}\pi_{x_4})\},$$

where

$$D_3 = (\eta_{x_1}\eta_{y_2} - \eta_{y_1}\eta_{x_2}) - \frac{1}{C}(\pi_{x_1}\eta_{x_2} - \eta_{x_1}\pi_{x_2}) - C(\pi_{y_1}\eta_{y_2} - \eta_{y_1}\pi_{y_2})$$

$$+ (\pi_{x_1}\pi_{y_2} - \pi_{y_1}\pi_{x_2}).$$

In this case the dominant reflection coefficients are $_3R_1$ and $_4R_2$. The coefficients $_3R_2$, $_4R_1$ are usually negligible.

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